Distance and Points of Closest Approach for Track Pairs in the MIPP Experiment

Abstract

We present a method for obtaining the distance of closest approach of two trajectories in three-dimensional space, and the point on each trajectory that is closest to the other (the points of closest approach). First, calculations are done for straight lines, and then for curved lines. The distance and points of closest approach, in particular, are useful for finding a vertex for two particle tracks in the MIPP detector at Fermi National Accelerator Lab.

Introduction

In the MIPP experiment, a beam usually made of protons, pions and kaons collides with an elemental target. The information provided for a particle track resulting from this collision includes a given position on the track (adequately close to, if not, the first measured point), the momentum vector at that point, the magnetic field acting on the particle and the particle charge. Using this information, the distance and points of closest approach between two of these tracks, one positively charged and the other negatively charged, may be considered to determine if these tracks form a vertex (V_0).

Distance and Points of Closest Approach for Straight Tracks

The general overview of the process starts with defining equations describing the tracks. From these, a vector of the distance between the two tracks is formed by subtracting the negative track equation from the positive one. This distance vector is squared, and the derivative is taken. When the derivative is equal to zero, the distance of closest approach is smallest.

First, the two tracks will be assumed to have no curvature. The equations for the straight tracks are

$$rp = r_{0p} + up*tp \qquad [1]$$

$$rn = r_{0n} + un*tn \qquad [2]$$

with *rp* representing the position on the positively charged track and *rn* the position on the negatively charged one. (All parts of the equations ending in *p* refer to the positive track, and all ending in *n* refer to the negative track.) r_0 is the position vector of the given position, *u* is the unit vector in the direction of the track momentum, and *t* is a parameter describing position on the tracks. The distance vector between the two tracks is

$$d = rp - rn = r_{0p} - r_{0n} + up*tp - un*tn$$

From here on, $r = r_{0p} - r_{0n}$. The distance vector squared is

$$|d|^2 = r^2 + 2r*up*tp - 2r*un*tn - 2up*un*tp*tn + tp^2 + tn^2$$

The derivatives with respect to the *t*'s are

$\partial|d|^2/\partial tp = 0 = 2r*up - 2up*un*tn + 2tp$
$\partial|d|^2/\partial tn = 0 = 2r*un - 2up*un*tp + 2tn$

The derivatives rewritten in canonical form are

$\partial|d|^2/\partial tp = tp - (up*un)*tn = -r*up$
$\partial|d|^2/\partial tn = -(up*un)*tp + tn = r*un$

The solutions of the linear system are

$tp = [-r*up + (r*un)(up*un)] / [1 - (up*un)^2]$
$tn = [r*un - (r*up)(up*un)] / [1 - (up*un)^2]$

Now *tp* and *tn* can be substituted into equations 1 and 2 respectively to obtain the point on each line that is closest to the other.

Points of Closest Approach for Curved Tracks

Now, the two tracks will be assumed to have curvature. The equations for the tracks now include a term describing the curve.

$rp = r_{0p} + up \cdot tp + cp \cdot tp^2$

$rn = r_{0n} + un \cdot tn + cn \cdot tn^2$

$c = k*(u \times \hat{y}) / |u \times \hat{y}|$ (the unit vector perpendicular to both the momentum and y-axis multiplied by *k*, a parameter describing the curve). We have chosen the magnetic field from the magnets on the outside of the chamber to be in the y-direction, and the trajectory will curve perpendicular to the field and the particle's velocity. The distance vector between the two tracks is
$d = rp - rn = r + up*tp - un*tn + cp*tp^2 - cn*tn^2$.
The distance vector squared is

$|d|^2 = r^2 + 2r*up*tp - 2r*un*tn + 2r*cp*tp^2 - 2r*cn*tn^2 - 2up*un*tp*tn + tp^2 + tn^2 - 2un*cp*tn*tp^2 - 2up*cn*tp*tn^2 - 2cp*cn*tp^2tn^2 + cp^2tp^4 + cn^2tn^4$

Terms that include $cp*up$ and $cn*un$ are zero because c is perpendicular to u. t is small because r_{0p} and r_{0n} are assumed to be close to the vertex, so we dropped terms that were 3rd t degree and higher. Taking the derivatives with respect to the t's, the result is

$\partial|d|^2/\partial tp = 0 = 2r*up + 4r*cp*tp - 2up*un*tn + 2tp - 4un*cp*tn*tp - 2up*cn*tn^2$
$\partial|d|^2/\partial tn = 0 = 2r*un + 4r*cn*tn - 2up*un*tp + 2tn - 4up*cn*tp*tn - 2un*cp*tp^2$

The expressions written in canonical form are

$\partial|d|^2/\partial tp = (1 + 2r*cp - 2un*cp*tn)*tp - (up*un - up*cn*tn)*tn = -r*up$
$\partial|d|^2/\partial tn = -(up*un + un*cp*tp)*tp + (1 - 2r*cn - 2up*cn*tp)*tn = r*un$

We calculated tp and tn iteratively, so for $tp0$ and $tn0$, we dropped the 2nd degree t terms:

$$(1 + 2r*cp)*tp0 - (up*un)*tn0 = -r*up$$
$$-(up*un)*tp0 + (1 - 2r*cn)*tn0 = r*un$$

Solving the linear system, we got

$$tp0 = \frac{-(r*up)(1 - 2r*cn) + (up*un)(r*un)}{(1 + 2r*cp)(1 - 2r*cn) - (up*un)^2}$$
$$tn0 = \frac{(r*un)(1 + 2r*cp) - (up*un)(r*up)}{(1 + 2r*cp)(1 - 2r*cn) - (up*un)^2}$$

Please note that when the tracks' curves are zero, $tp0$ and $tn0$ are reduced to tp and tn from the solution for the straight line tracks.

We defined $tp1 = tp0 + ep$ and $tn1 = tn0 + en$. We substituted $tp1$ and $tn1$ into the derivative expressions (including the 2nd degree terms):

$$(1 + 2r*cp)*tp1 - (up*un)*tn1 = -r*up + 2un*cp*tp0*tn0 + up*cn*tn0^2$$
$$-(up*un)*tp1 + (1 - 2r*cn)*tn1 = r*un + 2up*cn*tp0*tn0 + un*cp*tp0^2$$

The $tp0$ and $tn0$ terms that are also in the derivative expressions were cancelled, and we were left with

$$(1 + 2r*cp)*ep - (up*un)*en = 2un*cp*tp0*tn0 + up*cn*tn0^2 = sp$$
$$-(up*un)*ep + (1 - 2r*cn)*en = 2up*cn*tp0*tn0 + un*cp*tp0^2 = sn$$

We defined sp and sn to be the right sides of the previous expressions. After solving the linear system, we get

$$ep = \frac{sp*(1 - 2r*cn) + sn*(up*un)}{D}$$
$$en = \frac{sn*(1 + 2r*cp) + sp*(up*un)}{D}$$

$D = (1 + 2r*cp)(1 - 2r*cn) - (up*un)^2$, the denominator from the $tp0$ and $tn0$ expressions.

The radius of curvature, R, must be known in order find the curvature, k. R can be found from the relation $q*B_y*R = (p_x^2 + p_z^2)^{1/2}$, where q is the particle charge, B is the magnetic field (only in the y-direction as noted earlier), and p is the momentum of the particle. The 2nd derivative of a parabolic function is equated to the 2nd derivative of a circle equation with the radius as R.

$y = ax^2 \Rightarrow y'' = 2a$

$x^2 + y^2 = R^2 \Rightarrow y'' = R^2(R^2 - x^2)^{-3/2}$

Estimating the curvature at $x = 0$ of a parabola and a circle with radius R to be equal, the result is

$y'' = 1/R = 2a \Rightarrow a = 1/(2R) = k$.

After plugging t_{p1} into *rp* and t_{n1} into *rn*, the x-, y- and z-positions of the points on the tracks that begin and end the distance of closest approach line segment can be found by using the respective components of the vectors used in *rp* and *rn* and solving for the respective components of *rp* and *rn*. Using these coordinates for each line, the distance between them can be calculated, and this will be the distance of closest approach. The midpoint on this segment will be the point of closest approach.

Conclusion

The method we presented is a reasonably accurate way of finding the points of closest approach on two trajectories only knowing the